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Radiative characteristics of beds made of large spheres containing an absorbing and scattering medium

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Abstract

The aim of this paper is to characterize the extinction coefficient, the scattering albedo and the scattering phase function of beds made of spherical particles containing an absorbing and scattering medium. The radiative properties of the medium contained in the particles are assumed to be known. Moreover, the particles in the bed are large compared to the wavelength considered and are supposed to be sufficiently distant from each other to scatter radiation independently. Thus, the characteristics of the whole bed can be determined from the properties of the particles alone. These properties are computed using a Monte Carlo procedure which simulates the interaction of one particle with a plane incident wave. The characteristics of the whole bed are determined using the independent scattering theory by adding the contributions of each particle. The evolutions of these characteristics with the properties of the inside medium, the porosity of the bed and the radius of the particles are investigated.

Keywords: Radiative properties; Particle bed; Independent scattering; Absorption; Scattering

1. Introduction

The modeling of radiative transfer in particulate media is of primary importance for the prediction and optimization of the insulating capabilities of such materials in many engineering applications. An extensive review of radiative transfer in dispersed media was carried out by Viskanta and Mengüç in 1989 [1], and by Baillis and Sacadura in 2000 [2]. Some examples of these materials include packed beds.

In order to evaluate radiative heat transfer, radiative characteristics are required in conjunction with the integro-differential RTE. These characteristics are: the extinction coefficient β, the scattering albedo ω and the scattering phase function $P(\theta)$. Radiative transfer prediction then depends on the accuracy not only on the radiative characteristics values but also on the radiative transfer modeling techniques. In particulate media, in order to predict those characteristics, it is necessary to know how the particles individually interact with incident radiation upon them.

The question concerning the interaction of radiation with a particle can be solved by considering a plane, monochromatic wave incident upon a particle of given shape and size. This has been done for spheres of given optical constants by resolving the Maxwell's equation with the Mie theory [3–7]. Moreover, numerous studies have already been made to model the radiative transfer in beds of spherical particles and which deal with independent or dependent scattering and the limits of application of the theory of independent scattering [2]. Several of these studies use the hypothesis of independent scattering which assumes that the interaction of one particle in the bed with the radiation field is not influenced by the presence of other particles. As an example, we can cite the works of Dombrovsky [8] on semi-transparent particles, and Fedorov [9] on glass foams made of air bubbles in a semi-transparent matrix. Other researchers have

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Nomenclature

a_n	nth Legendre coefficient	
D	random generated number	
dist	distance travelled by the ray in the inside medium before interception	m
$d\omega(\theta_i')$	the solid angle around direction θ_i'	Sr
dx, dy, dz	directing cosines of the ray's direction on the different axis	
n_{max}	order of the expansion in Legendre polynomials	
N_{part}	the number of particles per unit volume of the bed	m^{-3}
$P(\theta)$	scattering phase function	
Q	extinction efficiency factor	
R	radius	m
S_{part}	the cross section of the particles	m^2
x, y, z	coordinates of the ray on the different axis	m

Greek symbols

λ	wavelength	m
β	extinction coefficient	m^{-1}
ω	scattering albedo	
ε	porosity	
θ	scattering angle	rad
θ'	angle between the starting ray and the ray scattered away from the sphere	rad
ϕ	azimuthal scattering angle	rad
ν	size parameter of the particle, $=\frac{2\pi R_{part}}{\lambda}$	

Subscripts

bed	of the bed
part	of the particle
ind	from independent scattering theory
init	initial
ins	of the inside medium
leg	Legendre polynomial
new	new value

tried to take into account the dependent effect in beds made of opaque or semi-transparent particles. Most of these authors used ray-tracing approaches given that they permit to take into account the influence of the neighbouring particles on the radiation field arriving on a particle. We can cite the work of Chan and Tien [10], Kaviany and Singh [11,12], Kamiuto [13] and Coquard et al. [14].

All these studies were interested in beds made of spherical particles containing a non-scattering medium with given optical constants and only few works have dealt with radiative properties of spherical particles containing an absorbing and scattering medium. Mishchenko and Macke [15] used a method combining ray tracing and the Monte Carlo (RTMC) technique to compute the asymmetry parameters for isolated spherical particles filled with large numbers of small scattering inclusions. However, they were only interested in the phase function of the particle and have not studied the evolution of the radiative characteristics of the particle with the radiative properties of the inside medium. Sherma and Jones [16,17] used the technique developed by Mishchenko and Macke to validate an approximate formula designed to compute the absorption and scattering efficiencies of spherical particles containing spherical inclusions. Nevertheless, the inclusions were considered purely absorbing.

In this study, we investigate all the radiative characteristics of media made of the arrangement of spherical particles containing an absorbing and also scattering medium whose properties are known. The evolution of these characteristics is studied. In the beds considered, the particles are supposed to be sufficiently distant from each other to scatter radiation independently. The results obtained could be interesting in a number of applications such as the radiative heat transfer in PSE foams, the radio-wave propagation in urban settings or microwave propagation in anechoic chambers.

Firstly, we focus on radiative characteristics of one particle alone using a Monte Carlo procedure which simulates a plane incident wave on the particle. The influences of the radiative properties of the medium contained in the particles and of the radius of the particles on the particle radiative characteristics are discussed. Moreover, a special attention is put to study if there are simple relations between inside radiative characteristics and those of the bed.

Thereafter, the radiative characteristics of the whole beds are calculated using the independent scattering hypothesis. The evolutions of the extinction coefficient, scattering albedo and phase function with the porosity, the radius of the particles and the properties of the inside medium are quantitatively studied.

We precise that a pioneer work on the evolution of the radiative properties of beds made of opaque, absorbing and reflecting spherical particles [8], which uses a similar Monte Carlo technique, has already been conducted. The results of the model developed were validated by comparing the radiative properties obtained by this model with correlations given by different authors, and by comparing the transmittances measured by Chen and Churchill on beds of metallic spherical spheres of given thickness with the theoretical results obtained using our method. The accordance proves to be quite good.

2. Hypotheses of the study

First, we assume that the radius R_{part} of the spherical particles of the beds is much greater than the wavelength λ of the thermal radiation studied. For a radiative heat transfer calculation at 300 K, most of the wavelength concerned ranges between 5 µm and 50 µm. We will consider beds made of particles with radius greater than 100 µm. As a result, the size parameter ν of such particles is much greater than 1 and then, the radiation/matter interaction can be treated using the geometric optics laws.

Moreover, the influence of diffraction on the radiative heat transfer is neglected. Indeed, the diffraction phase function for particles with a large size parameter ν is predominantly oriented in the forward direction and diffracted rays are very close to transmitted rays. We then assume, here, that diffraction can be treated as transmission.

The extinction coefficient β_{ins}, the scattering albedo ω_{ins} and the phase function $P_{\text{ins}}(\theta)$ of the medium contained in the spherical particles are constant and homogeneous and are supposed to be known. The scattering properties of the medium contained in the spherical particles are independent of the azimuth angle (azimuthal symmetry) and the phase function is expressed in the form of an expansion in Legendre polynomials. Then we have:

$$P_{\text{ins}}(\theta) = \sum_{n=0}^{n_{\max}} a_n P_{\text{leg}}^n(\theta)$$

Given that the medium is made of spherical particles and that the scattering properties of the inside medium are independent of azimuth angle, we assume that all the scattering properties of the bed are independent on the azimuth angle (azimuthal symmetry).

Finally, we will also consider that the refractive index of the inside medium is equal to unity and, therefore, no reflection occurs at the interface between the air and the spherical particles.

3. Radiative characteristics of spherical particles alone

3.1. Description of the ray-tracing method for one particle

Interaction of light with a spherical particle can be described by three radiative characteristics. These characteristics are:

- the extinction efficiency factor Q_{part} corresponding to the fraction of incident radiant energy that is absorbed or scattered by the particle;
- the albedo ω_{part} corresponding to the fraction of intercepted energy that is scattered away from the particle;
- the scattering phase function $P_{\text{part}}(\theta)$ which describes the angular distribution of the energy scattered away from the sphere.

We have developed a method based on a 3-D Monte Carlo procedure in order to determine these characteristics for spherical particle containing an absorbing and scattering medium. This method simulates the interaction of a plane incident wave with the particle. The plane incident wave is divided in light rays of unit energy which are uniformly spread on the enter face of the spherical particle (see Fig. 1). The path of each ray is analyzed by the 3-D procedure in the Cartesian coordinate system x, y, z centered on the center of the particle:

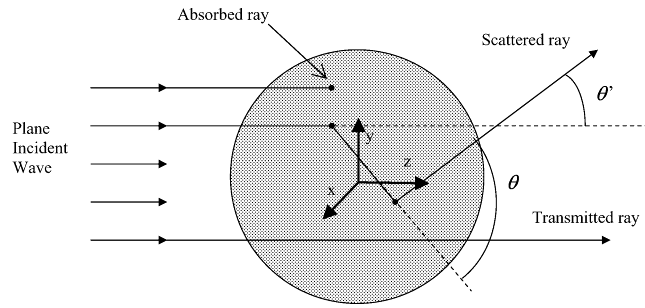

Fig. 1. Illustration of the interaction of a plane incident wave with the spherical particle.

(1) The starting coordinates x_{init} and y_{init} of this ray are randomly chosen using random generated numbers D_i $(0 < D_i < 1)$ by:

$$x_{\text{init}} = (D_1 - 0.5) \times 2 \times R_{\text{part}}$$
$$y_{\text{init}} = (D_2 - 0.5) \times 2 \times R_{\text{part}}$$

(2) If $\sqrt{x_{\text{init}}^2 + y_{\text{init}}^2} > R_{\text{part}}$, the ray will not hit the particle. Operation (1) is then repeated until the ray hits the sphere.

(3) The direction of the plane incident wave is arbitrarily chosen along the z-axis. Then, the starting directions of the generated ray are set to:

$$\mathrm{d}x = 0, \qquad \mathrm{d}y = 0, \qquad \mathrm{d}z = 1$$

(4) The procedure determines the coordinate x, y, z of the point where the ray hits the particle. We have: $x = x_{\text{init}}$, $y = y_{\text{init}}$ and $z = -\sqrt{R^2 - y_{\text{init}}^2 - x_{\text{init}}^2}$.

(5) The distance *dist* traveled by the rays in the particle before being intercepted is computed from the extinction coefficient β_{ins} and from a random generated number D by:

$$dist = \frac{-Ln(D)}{\beta_{\text{ins}}}$$

(6) The coordinate of the point where the ray is intercepted are calculated by

$$x_{\text{new}} = x + dist \times \mathrm{d}x$$
$$y_{\text{new}} = y + dist \times \mathrm{d}y$$
$$z_{\text{new}} = z + dist \times \mathrm{d}z$$

The procedure continues with the new values $x = x_{\text{new}}$, $y = y_{\text{new}}$, $z = z_{\text{new}}$.

- If $\sqrt{x^2 + y^2 + z^2} > R_{\text{part}}$, the ray has left the sphere:
  - If it has never been scattered by the inside medium, the ray is transmitted through the particle (transmitted ray). The parameter *ntrans* is incremented.

The procedure for the current ray stops and it starts again from operation (1) with a new starting ray. The parameter *compt* counting the number of rays generated is incremented.
  - If it has already been scattered one or several times, the procedure calculates the angle θ' between the incident and leaving direction. We have: $\theta' = \cos^{-1}(\mathrm{d}z)$. The parameter $S(\theta')$ is incremented. The procedure for the current ray stops and it starts again from operation (1) with a new starting ray. The parameter *compt* is incremented.
- If $\sqrt{x^2+y^2+z^2} < R_{\text{part}}$, the ray is still in the particle. The procedure continues and goes to operation (7).

(7) The procedure determines whether the ray is scattered or absorbed by the inside medium using a random generated number D.

• If $D > \omega_{\text{ins}}$, the ray is absorbed by the inside medium. The parameter *Abso* is incremented. The procedure for the current ray stops and it starts again from operation (1) with a new starting ray. The parameter *compt* is incremented.
• If $D < \omega_{\text{ins}}$, the ray is scattered by the inside medium. The choice of the scattering angle θ is dictated by the scattering phase function of the inside medium. Indeed, the probability for the incident ray to be scattered in a direction making an angle α with the incident direction is equal to: $\frac{1}{2}P_{\text{ins}}(\alpha)\sin(\alpha)\,\mathrm{d}\alpha$. The scattering angle θ is then determined from the scattering phase function $P_{\text{ins}}(\theta)$ using random numbers D $(0 < D < 1)$ by computing the value of θ which satisfies the relation:

$$\frac{\int_0^{\theta} P_{\text{ins}}(\alpha)\sin(\alpha)\,\mathrm{d}\alpha}{\int_0^{\pi} P_{\text{ins}}(\alpha)\sin(\alpha)\,\mathrm{d}\alpha} = D$$

The values of the integrals are determined analytically whereas the value of θ is identified by dichotomy. As the inside medium scatters with azimuthal symmetry, the azimuthal scattering angle ϕ is randomly chosen using a random number D: $\phi = 2\pi \times D$. The procedure then determines the new direction $\mathrm{d}x_{\text{new}}$, $\mathrm{d}y_{\text{new}}$ and $\mathrm{d}z_{\text{new}}$ of the ray from the former direction $\mathrm{d}x$, $\mathrm{d}y$, $\mathrm{d}z$ and from the angles θ and ϕ. After several changes in the coordinate system, we have:

– If $\mathrm{d}y > 0$,

$$\begin{cases} \mathrm{d}x_{\text{new}} = \sin(\theta)\sin(\phi)\cos(\sin^{-1}(\mathrm{d}x)) \\ \qquad + \sin(\theta)\cos(\phi) \times \mathrm{d}z \times \mathrm{d}x \\ \qquad + \cos(\theta)\sin(\cos^{-1}(\mathrm{d}z)) \times \mathrm{d}x \\ \mathrm{d}y_{\text{new}} = -\sin(\theta)\sin(\phi) \times \mathrm{d}x \\ \qquad + \sin(\theta)\cos(\phi) \times \mathrm{d}z \times \cos(\sin^{-1}(\mathrm{d}x)) \\ \qquad + \cos(\theta)\sin(\cos^{-1}(\mathrm{d}z))\cos(\sin^{-1}(\mathrm{d}x)) \\ \mathrm{d}z_{\text{new}} = \cos(\theta) \times \mathrm{d}z \\ \qquad - \sin(\theta)\cos(\phi)\sin(\cos^{-1}(\mathrm{d}z)) \end{cases}$$

– If $\mathrm{d}y < 0$,

$$\begin{cases} \mathrm{d}x_{\text{new}} = -\sin(\theta)\sin(\phi)\cos(\sin^{-1}(\mathrm{d}x)) \\ \qquad + \sin(\theta)\cos(\phi) \times \mathrm{d}z \times \mathrm{d}x \\ \qquad + \cos(\theta)\sin(\cos^{-1}(\mathrm{d}z)) \times \mathrm{d}x \\ \mathrm{d}y_{\text{new}} = -\sin(\theta)\sin(\phi) \times \mathrm{d}x \\ \qquad - \sin(\theta)\cos(\phi) \times \mathrm{d}z \times \cos(\sin^{-1}(\mathrm{d}x)) \\ \qquad - \cos(\theta)\sin(\cos^{-1}(\mathrm{d}z))\cos(\sin^{-1}(\mathrm{d}x)) \\ \mathrm{d}z_{\text{new}} = \cos(\theta) \times \mathrm{d}z \\ \qquad - \sin(\theta)\cos(\phi)\sin(\cos^{-1}(\mathrm{d}z)) \end{cases}$$

Then, the procedure goes back to operation (5) with the new values of $\mathrm{d}x = \mathrm{d}x_{\text{new}}$, $\mathrm{d}y = \mathrm{d}y_{\text{new}}$ and $\mathrm{d}z = \mathrm{d}z_{\text{new}}$. The procedure stops when the number of rays generated *compt* reaches a large fixed value *ntirs*. The procedure is summarized by the diagram on Fig. 2.

For a great number of rays generated, we can compute:
- $Trans = \frac{ntrans}{ntirs}$: the proportion of the starting energy that pass through the particle without being scattered or absorbed (transmitted rays);
- $Sca = \frac{ntirs - Abso - ntrans}{ntirs}$: the proportion of starting energy leaving the particle after one or several scattering by the inside medium (scattered rays);
- $Abs = \frac{Abso}{ntirs}$: proportion of starting energy absorbed by the particle;
- $E(\theta') = \frac{S(\theta')}{\int_0^{180} S(\theta')\,\mathrm{d}\theta'}$: the average angular repartition of the energy scattered outside the particle. In prac-

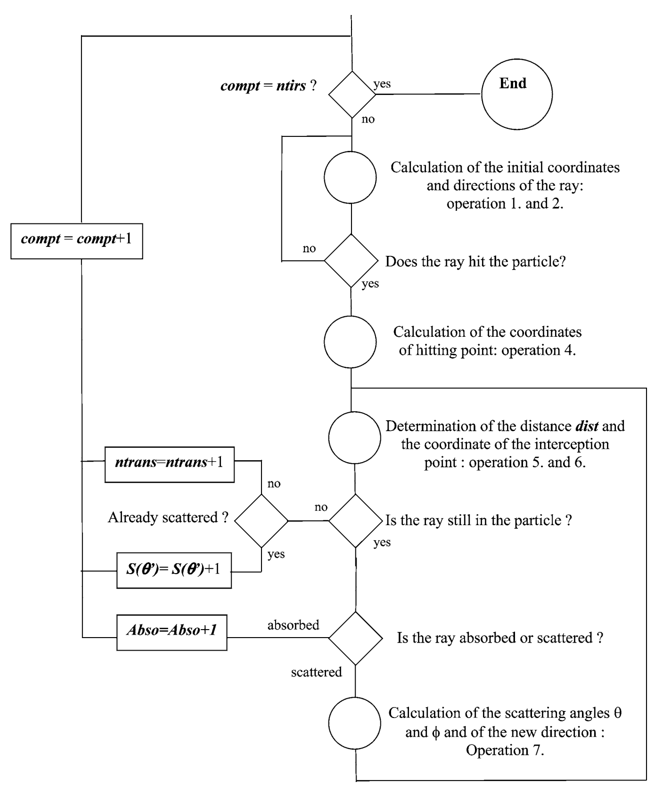

Fig. 2. Diagram for the Monte Carlo procedure simulating the interaction of the plane incident wave with the particle.

tice, the repartition is discretized for $\theta_0 = 0°$, $\theta_1 = 1°$, ..., $\theta_{180} = 180°$. Then, all the rays leaving the sphere with an angle θ' comprised between $\theta'_i - 0.5°$ and $\theta'_i + 0.5°$ are regrouped in $S(\theta'_i)$.

The radiative characteristics of the spherical particle are then calculated by:

$$Q_{\text{part}} = (1 - \mathit{Trans})$$
$$\omega_{\text{part}} = \frac{\mathit{Sca}}{(1 - \mathit{Trans})}$$
$$P_{\text{part}}(\theta') = \frac{E(\theta')\,\mathrm{d}\Omega}{\int_{\Omega=4\pi} E(\theta')\,\mathrm{d}\Omega}$$

Usually, all the rays hitting a particle are reflected, refracted or absorbed, and then, if diffraction is neglected, we have $Q_{\text{part}} = 1$ ($Q_{\text{part}} = 2$ if diffraction phenomena are taken into account which constitutes the well-known "extinction paradox"). However, in our case, given that the inside medium is not opaque, some rays can pass through the particle without being refracted, absorbed nor scattered and then $0 < Q_{\text{part}} \leqslant 1$.

3.2. Evolution of the radiative characteristics of one particle

In order to characterize the influence of the radiative characteristics of the inside medium on the particle's radiative characteristics, the ray-tracing method have been applied to different particles. For every cases, the number of rays generated during the Monte Carlo procedure is $\mathit{ntirs} = 10^6$.

The calculations tend to show that dimensionless number $\beta_{\text{ins}} R_{\text{part}}$ is of great importance. The evolution of the extinction efficiency, the scattering albedo and the scattering phase function of the ball is indeed strongly dependent on its value. We also observe that the way of evolution with $\beta_{\text{ins}} R_{\text{part}}$ is practically the same whatever ω_{ins} and $P_{\text{ins}}(\theta)$ are.

As regards the extinction efficiency factor Q_{part}, it only depends on the value of $\beta_{\text{ins}} R_{\text{part}}$. Its evolution is depicted on Fig. 3. The convergence of Q_{part} has been estimated from the amplitude of the fluctuations of its value for several simulations using different random numbers. The fluctuations of the results proves to be always lower than 0.1% when $\mathit{ntirs} = 10^6$.

As can be seen, the evolution of Q_{part} is close to $(1 - e^{-\beta_{\text{ins}} R_{\text{part}}})$ and then tends to the asymptote $Q_{\text{part}} = 1$ when $\beta_{\text{ins}} R_{\text{part}}$ reaches large values. Most of the incident energy is then intercepted. This result is not surprising given that, in semi-transparent medium the intensity decreases exponentially when the extinction coefficient grows. This evolution is of course independent on ω_{ins} and $P_{\text{ins}}(\theta)$.

The variations of the scattering albedo ω_{part} of the spherical particle with $\beta_{\text{ins}} R_{\text{part}}$ are shown on Fig. 4 for different cases. Its value also depends on the albedo and the phase function of the inside medium. The fluctuations on the value of the albedo has been estimated by the same manner as the extinction efficiency factor and turns out to be always lower than 0.1% when $\mathit{ntirs} = 10^6$.

For small $\beta_{\text{ins}} R_{\text{part}}$ values, ω_{part} is close to ω_{ins}. Indeed, in this case, most of the rays that are scattered out of the sphere are scattered only one time and then corresponds to the single scattering albedo of the inside medium.

On the other hand, when $\beta_{\text{ins}} R_{\text{part}}$ is large enough, the ω_{part} can be noticeably smaller than ω_{ins}. For example, when $\beta_{\text{ins}} R_{\text{part}} = 8$ and $\omega_{\text{ins}} = 0.9$, the albedo of the sphere is 0.57 for an isotropically scattering inside medium. It can be explained by the fact that when $\beta_{\text{ins}} R_{\text{part}}$ is large, the rays that go out the sphere undergo several scattering and then cover a long distance before leaving the volume. The fraction of intercepted energy which has been absorbed by the particle is then more significant than when the rays are scattered a small number of time. So, the fraction energy that is scattered away from the sphere becomes less and less important when $\beta_{\text{ins}} R_{\text{part}}$ values increase. This observation is confirmed by the calculations since Fig. 4 shows a monotone decrease of ω_{part} with $\beta_{\text{ins}} R_{\text{part}}$ in all cases.

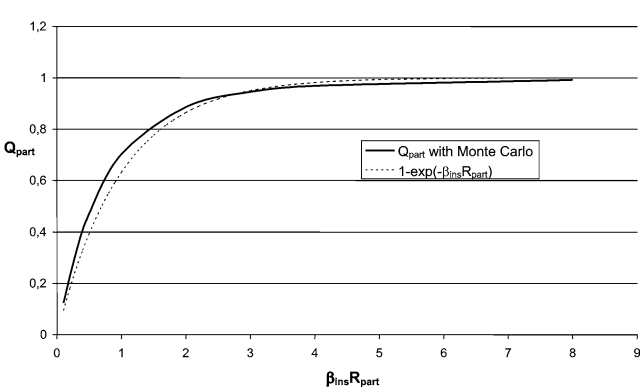

Fig. 3. Evolution of the extinction efficiency factor of the particle with the dimensionless number $\beta_{\text{ins}} R_{\text{part}}$.

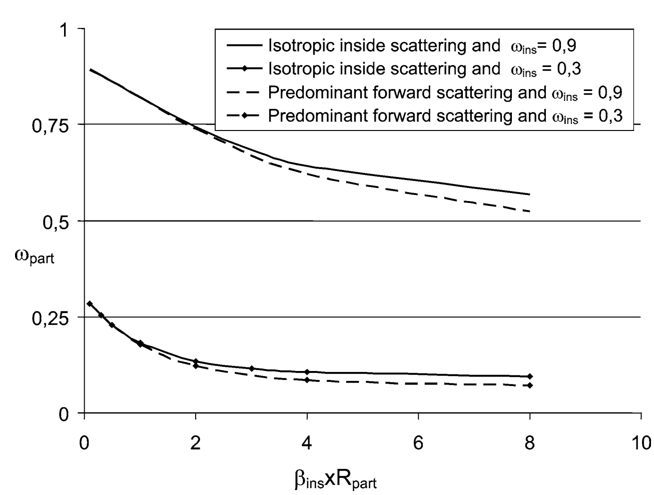

Fig. 4. Evolution of the particle's scattering albedo with the dimensionless number $\beta_{\text{ins}} R_{\text{part}}$ for different inside medium.

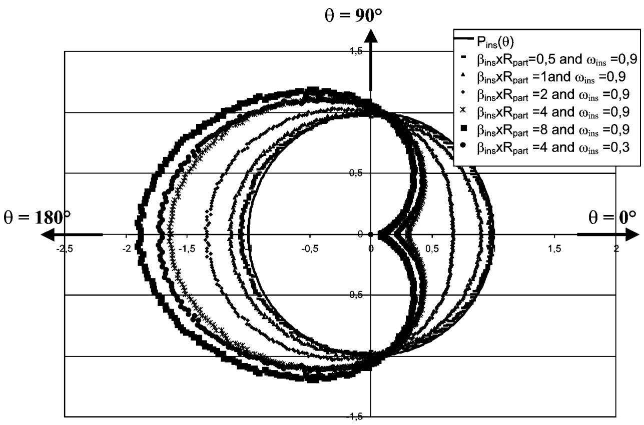

Fig. 5. Evolution of the scattering phase function of the particle $P_{\text{part}}(\theta)$ with the dimensionless number $\beta_{\text{ins}}R_{\text{part}}$ for an isotropically scattering inside medium.

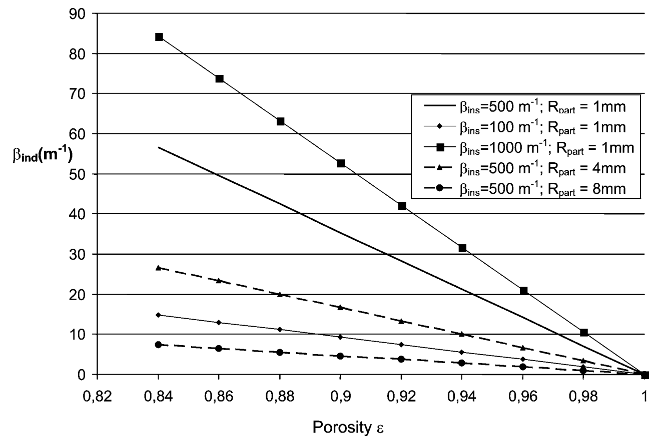

Fig. 6. Evolution of the bed's extinction coefficient with the porosity for different radius of the particles and different extinction coefficient of the inside medium.

The scattering phase function $P_{\text{ins}}(\theta)$ of the inside medium also have a slight influence on ω_{part} as shown on Fig. 4. For a predominant forward scattering phase function of the inside medium ($P_{\text{ins}}(\theta) = 1 - \cos(\theta)$) the decrease of ω_{part} with $\beta_{\text{ins}}R_{\text{part}}$ is more pronounced in the two cases ($\omega_{\text{ins}} = 0.9$ and $\omega_{\text{ins}} = 0.3$). Indeed, given that, for large values of $\beta_{\text{ins}}R_{\text{part}}$, the first scattering of the rays takes place near the entering face, the path covered by the rays before leaving the sphere volume is longer when the rays are predominantly scattered in the forward direction. The fraction of energy that leaves the sphere without being absorbed is then less important.

As regards the scattering phase function of the particle $P_{\text{part}}(\theta)$, the value of $\beta_{\text{ins}}R_{\text{part}}$ is also of great importance. The evolution of $P_{\text{part}}(\theta)$ with $\beta_{\text{ins}}R_{\text{part}}$ is depicted in Fig. 5 for an isotropically scattering inside medium. The results show that $P_{\text{part}}(\theta)$ could be strongly different from $P_{\text{ins}}(\theta)$ when $\beta_{\text{ins}}R_{\text{part}}$ is large, as for the particle's albedo. It seems that the spherical shape of the particle favours backward scattering as shown on Fig. 5. $P_{\text{part}}(\theta)$ is then no longer isotropic as soon as $\beta_{\text{ins}}R_{\text{part}}$ exceed 0.5 and the sphere scatters backward preferably. Indeed, the length of penetration of one ray in the ball before being scattered is very small and the path length of a ray leaving the ball in a forward direction is therefore longer than that of a ray directly back-scattered outside of the sphere. The amount of energy leaving the sphere in forward directions is then smaller. We notice that this "back-scattering effect" appears whatever the phase function of the inside medium is.

For very large values of $\beta_{\text{ins}}R_{\text{part}}$ (> 8) $P_{\text{part}}(\theta)$ tend to the phase function of a diffusely opaque reflecting sphere which is characterized by no scattering in the incident direction. The length of penetration of a ray in the sphere is then so small that the only way for a ray to go out of the sphere without being importantly absorbed is to be directly scattered out in a back direction. Therefore, the particles behave like a sphere which reflects the radiant energy with the same probability for all directions (diffuse).

ω_{ins} also has an influence on $P_{\text{part}}(\theta)$ as shown on Fig. 5. For small values of ω_{ins}, the "back-scattering effect" due to the spherical shape of the particle is more pronounced than for larger values. Indeed, the energy of the rays scattered away from the sphere in forward directions and which leaves the ball after several inside scattering is much smaller for small ω_{ins} than for great ω_{ins}. Then, it also favors back-scattering.

4. Radiative characteristics of the whole bed

When independent scattering occurs, the extinction coefficient as well as the scattering albedo and phase function of the whole bed can be simply determined from the contributions of each particle. We then have:

$$\beta_{\text{ind}} = N_{\text{part}} \times S_{\text{part}} \times Q_{\text{part}}$$

$$\omega_{\text{ind}} = \omega_{\text{part}} \quad \text{and} \quad P_{\text{ind}}(\theta) = P_{\text{part}}(\theta)$$

For spherical particles, we have:

$$N_{\text{part}} = \frac{1-\varepsilon}{\frac{4}{3}\pi R_{\text{part}}^3}, \qquad S_{\text{part}} = \pi R_{\text{part}}^2$$

and then

$$\beta_{\text{ind}} = \frac{0.75 \times (1-\varepsilon)}{R_{\text{part}}} Q_{\text{part}}$$

We have represented on Fig. 6 the evolution of this extinction coefficient with the porosity for beds containing particles with different radius R_{part} and different extinction coefficient β_{ins}.

The extinction coefficient of the whole bed is linearly dependent on $(1 - \varepsilon)$. The dependency factor varies with β_{ins} and R_{part}.

We also have depicted on Fig. 7 the variation of β_{ind} with R_{part} for different values of β_{ins} for a bed with $\varepsilon = 0.95$.

We remark that the greater is R_{part}, the lower is the extinction coefficient of the whole bed, whatever the values of β_{ins} is. We also observe logically that the greater is β_{ins}, the

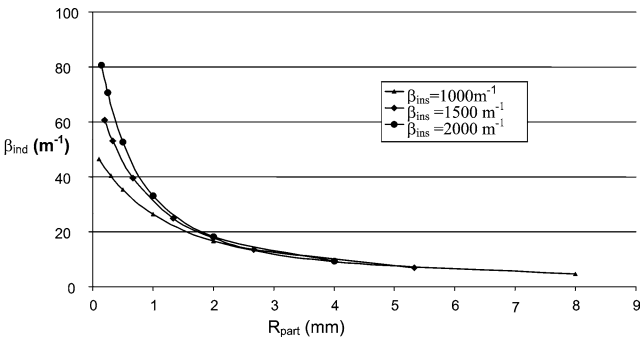

Fig. 7. Evolution of the extinction coefficient of the bed with the radius of the particles for different extinction coefficient of the inside medium.

more important is β_{ind}. Indeed, as was said before, the variation of the extinction efficiency factor Q_{part} are close to $1 - e^{-\beta_{\text{ins}} R_{\text{part}}}$. Then, we have:

$$\beta_{\text{ind}} \approx \frac{0.75 \times (1-\varepsilon) \times (1 - e^{-\beta_{\text{ins}} R_{\text{part}}})}{R_{\text{part}}}$$

For large values of $\beta_{\text{ins}} R_{\text{part}}$ (> 2), β_{ind} is practically independent on β_{ins}, given that $Q_{\text{part}} \cong 1$. The variation of β_{ind} with R_{part} is then quasi-proportional to $1/R_{\text{part}}$.

As regards the scattering albedo and scattering phase function, the conclusions are the same as for the particle alone given that the scattering characteristics of the whole bed are equal to those of one particle alone. Thus, the albedo of the bed decreases when the dimensionless number $\beta_{\text{ins}} R_{\text{part}}$ increases and is always lower than the albedo of the inside medium. Moreover, the phase function of the bed favors back-scattering when compared to the phase function of the inside medium. This effect is more pronounced when $\beta_{\text{ins}} R_{\text{part}}$ is large.

5. Conclusions

The innovative part of this paper consists in determining the radiative characteristics of beds made of spherical particles containing an absorbing and scattering medium. The particles are assumed to scatter radiation independently and then, the characteristics of the whole bed are obtained from the properties of one particle alone. The computation of these properties is based on a Monte Carlo procedure that simulates the interaction of a plane incident wave on the particle.

The characteristics of the particle alone prove to be strongly dependent on the dimensionless number $\beta_{\text{ins}} R_{\text{part}}$ and on the properties of the inside medium. We notice that the scattering albedo and phase function of one particle could greatly differ from those of the inside medium. Especially, the albedo of one particle is always lower than that of the inside medium. Moreover, the phase function of the particle scatters backward in a more important manner than the phase function of the inside medium. In the case of an isotropically scattering inside medium, the phase function of the particles varies between the phase function of the inside medium and that of an opaque diffusely reflecting sphere when the non-unit number $\beta_{\text{ins}} R_{\text{part}}$ increases.

As regards the radiative characteristics of the whole bed, we conclude that the extinction coefficient is linearly dependent on $(1-\varepsilon)$. However, it varies in a complicated manner with the radius of the particles and with the extinction coefficient of the inside medium. We propose a formula to approximate it easily according to the porosity, the particle radius and β_{ins}. Given that the particles are supposed to scatter independently, the evolution of the scattering albedo and phase function of the bed is the same as for the particles alone.

We, thus, have characterized the radiative behavior of such beds whose porosity is sufficiently important to consider that the independent scattering theory is valid. In order to take into account the dependent scattering effect, more investigations are required.

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